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INVESTIGATION OF THE ALGEBRAIC SYSTEM OF INFINITE ORDER OCCURRING IN SOLVING THE PROBLEM FOR A SEMI-STRIP

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The algebraic system of equations of infinite order studied here occurs during the solution of the problem of the theory of elasticity concerning a symmetrically loaded semi-strip clamped at the one end. The system is solved using the iteration method. First, out of the matrix of the system a sub-matrix is selected, characterizing the behavior of the solution at large values of the index of the unknown. It is proved and confirmed by concrete examples, that the solution of the basic system differs little from the solution of a simplified system. An asymptotic expansion is obtained for the solution of the simplified system for the large values of the index of the unknown and an approximate method is given for the determination of its coefficients.

An infinite system of algebraic equations for a semi-strip with stress-free longitudinal edges and displacements specified at its end was discussed in [1] where it was proved that the system is completely regular. Earlier [2] the behavior of the solution at large values of the index of the unknown was explained in a different manner, when solving an axisymmetric three-dimensional problem for a rigidly clamped plate. Only the first term of the asymptotics was taken into account when concrete problems were solved.

1. We consider a symmetric problem for a semi-strip under the following boundary conditions [3]:

$$x = 0,\ u = 0,\ v = 0 \tag{1.1}$$

$$y = 1,\ \sigma_{y_1} = -P\delta\,(x - c)\ /\ h,\ \tau_{x_1 y_1} = 0 \tag{1.2}$$

Here u and v denote the displacements along the x_1- and y_1-axes, respectively, ($x_1 = xh$, $y_1 = yh$), while $\tau_{x_1 y_1}$ and σ_{y_1} denote the tangential and normal stresses (see Fig. 1). In solving the problem we use the fundamental relations of the plane theory of elasticity written in terms of displacements [3], and write u and v as a superposition of the solutions of the following two auxiliary problems:

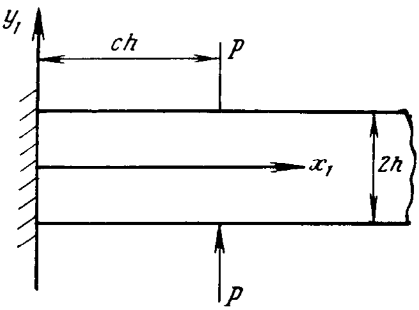

Fig. 1

1) the problem for a semi-strip with the boundary conditions ($\psi\,(x)$ is an unknown function)

$$x = 0,\ v\,(y) = 0,\ \sigma_{x_1} = 0,\ |\,y\,| < 1$$

$$y = 1,\ \tau_{x_1 y_1} = 0,\ \partial u\ /\ \partial y = \psi\,(x), \qquad 0 < x < \infty$$

2) the problem for a semi-strip, periodically continued into the region $|\,y\,| > 1$ with the boundary conditions

$$x = 0,\ \sigma_{x_1}\,(y) = \sigma\,(y),\ v\,(y) = 0$$

$$y = 1,\ \tau_{x_1 y_1} = 0,\ \partial u\ /\ \partial y = 0$$

Here $\psi\,(x)$ and $\sigma\,(y)$ are such that the boundary conditions (1.1) and (1.2) hold for the general problem.

A solution of (1) is given by

$$u_1 = \frac{2}{\pi}\int_0^\infty \left[(A_1 \operatorname{ch}\lambda y + A_2 \lambda y \operatorname{sh}\lambda y)\cos\lambda x + \frac{\psi_0(\lambda)(\nu - 1)}{\lambda^2(\nu + 1)}\right] d\lambda + C_1 \tag{1.3}$$

$$v_1 = \frac{2}{\pi}\int_0^\infty (B_1 \operatorname{sh}\lambda y + A_2 \lambda y \operatorname{ch}\lambda y)\sin\lambda x\, d\lambda$$

$$A_1 = B_1 - \theta\,\frac{\nu + 2}{2}, \qquad B_1 = -\frac{\psi_0(\lambda)}{\lambda \operatorname{sh}\lambda} + \nu\theta\,\frac{\lambda \operatorname{ch}\lambda}{2\operatorname{sh}\lambda}$$

$$\theta = \frac{-2\psi_0(\lambda)}{(\nu + 1)\lambda \operatorname{sh}\lambda}, \qquad A_2 = -\frac{\nu\theta}{2}, \qquad \nu = \frac{1}{1 - 2\mu}$$

$$\sigma_{y_1} = -\int_0^\infty \frac{\psi_0(\lambda)\,\Delta_+\, 2\nu G \sin\lambda x}{h\pi(\nu + 1)\operatorname{sh}^2\lambda}\, d\lambda$$

$$\Delta_+ = \operatorname{sh} 2\lambda + 2\lambda, \qquad \psi_0(\lambda) = \int_0^\infty \psi(t)\cos\lambda t\, dt$$

and a solution of (2) is given by

$$u_2 = C_2 + 2\sum_{n=1}^{\infty} u_n^{\circ}(x)\cos\omega_n y, \qquad \omega_n = \pi n \tag{1.4}$$

$$v_2 = 2\sum_{n=1}^{\infty} v_n^{\circ}(x)\sin\omega_n y$$

$$u_n^{\circ}(x) = -\exp(-\omega_n x)\frac{(\nu + 2 + \nu\omega_n x)\, h\sigma_n}{2(\nu+1)\,\omega_n G}$$

$$v_n^{\circ}(x) = -\exp(-\omega_n x)\frac{\nu x h\sigma_n}{2(\nu+1)\,\omega_n G}$$

$$\sigma_n = \int_0^1 \sigma(t)\cos\omega_n t\, dt$$

$$\sigma_{y_1} = 2\sum_{n=1}^{\infty}(-1)^n \exp(-\omega_n x)\,\sigma_n \frac{(\nu - 1 - \nu\omega_n x)}{(\nu+1)}$$

Here C_1 and C_2 are arbitrary constants, μ is the Poisson's ratio and G is the torsion modulus. Formulas (1.3) and (1.4) imply that the solution of the problem obtained by superposing the solutions of (1) and (2), satisfies the boundary conditions

$$x = 0, \ \sigma_{x_1} = \sigma(y), \ v(y) = 0$$
$$y = 1, \qquad \tau_{x_1 y_1} = 0$$

We further choose $\psi(x)$ so that the condition (1.2) holds for the normal stress. Let us substitute σ_{y_1} from (1.3) and (1.4) into (1.2) and apply the Fourier sine transform to the resulting expressions. This gives

$$-\frac{\psi_0(\lambda)\,\nu\Delta_+}{(\nu+1)\,\mathrm{sh}^2\lambda} - 2\sum_{m=1}^{\infty}(-1)^m \frac{h\sigma_m \lambda\varphi_m(\lambda)}{(\nu+1)\,G} = \tag{1.5}$$

$$\frac{h}{G}\int_0^{\infty} \sigma_{y_1}(t)\Big|_{y=1} \sin\lambda t\, dt = -\frac{P\sin\lambda c}{G}$$

$$\varphi_m(\lambda) = [(\nu+1)\,\omega_m^2 - (\nu-1)\,\lambda^2] \,/\, (\omega_m^2 + \lambda^2)^2$$

The function $\sigma(y)$ is defined in such a manner that the boundary condition (1.1) holds for the longitudinal displacement. To do this, we substitute (1.3) and (1.4) into (1.1) and apply the finite Fourier cosine transform over the variable y. Then

$$\sigma_k = \frac{4G(-1)^k}{\pi(\nu+2)\,h}\int_0^{\infty}\psi_0(\lambda)\,\omega_k\varphi_k(\lambda)\, d\lambda \tag{1.6}$$

$$C_1 + C_2 = 0, \qquad k = 0$$

Taking into account the relations connecting $\psi_0(\lambda)$ and $\psi(t)$ in (1.3), we transform (1.6) into

$$\sigma_k = \frac{2G(-1)^k}{(\nu+2)\,h}\int_0^{\infty}\psi(t)\exp(-\omega_k t)(1 + \nu\omega_k t)\, dt \tag{1.7}$$

Thus the obtaining of the solution of the problem is reduced to the determination of two functions $\psi(t)$ and σ_k using (1.5) and (1.7) or (1.5) and (1.6).

Let us now substitute (1.7) into (1.5) and, after some manipulations, apply to the expression obtained the inverse Fourier sine tranform over the variable λ. Having done this, we use the expression for $\psi_0(\lambda)$ appearing in (1.3) to construct an integral equation in $\psi(t)$. (Another method of constructing an analogous integral equation is given in [4]). Substituting $\psi_0(\lambda)$ obtained from (1.5) into (1.6), we obtain the following system of algebraic equations:

$$A_{(k)} = \sum_{m=1}^{\infty} M_{km} A_{(m)} + d_k, \qquad k \geqslant 1 \tag{1.8}$$

$$A_{(k)} = \sigma_k (-1)^k \frac{1/4\pi\nu(\nu+2)h}{P(\nu+1)}$$

$$M_{km} = -\frac{8}{\pi\nu(\nu+2)} \int_0^\infty \lambda\omega_k \varphi_k(\lambda)\varphi_m(\lambda) \operatorname{sh}^2\lambda \frac{d\lambda}{\Delta_+}$$

$$d_k = \int_0^\infty \omega_k \varphi_k(\lambda) \sin\lambda c \operatorname{sh}^2\lambda \frac{d\lambda}{\Delta_+}$$

2. Let us investigate the system (1.8) which can be written in the matrix form as

$$A = LA + d \tag{2.1}$$

$$A = (A_{(k)}), \; d = (d_k), \; L = (M_{km})$$

A unique bounded solution of (2.1) exists and can be found using the method of successive approximations. In fact, let us consider a Banach space R of all bounded sequences of real numbers Y_k with the norm

$$\|Y\| = \max_k \|Y_k\| \tag{2.2}$$

The condition that the operator L is bounded in R and the fact that $d \subset R$ together imply that the operator L exists in $D \subseteq R$. The condition of boundedness of the operator L has the form

$$\|LA\| \leqslant q\|A\| \tag{2.3}$$

From [1] follows

$$q < \frac{4(\mu-\mu^2)^{1/2}}{\pi(3-4\mu)} - \frac{1-2\mu}{3-4\mu}\left[1 - \frac{4}{\pi}\operatorname{arctg}(\mu^{-1}-1)^{1/2}\right] < 0.64 \tag{2.4}$$

i.e. $q < 1$. Thus L is bounded and is a compression operator.

Next we discuss another method of obtaining the solution of (2.1). Out of L in (2.1) we select an operator L_1 characterizing the behavior of L at large values of the indices k and m

$$L_1 = (E_{km}), \quad E_{km} = -\frac{4}{\pi\nu(\nu+2)} \int_0^\infty \lambda\omega_k\varphi_k(\lambda)\varphi_m(\lambda)\, d\lambda \tag{2.5}$$

In this case L has the form

$$L = L_1 + L_2, \; L_2 = (G_{km}) \tag{2.6}$$

$$G_{km} = \frac{4}{\pi\nu(\nu+2)} \int_0^\infty \lambda\omega_k\varphi_k(\lambda)\varphi_m(\lambda)\psi_1(\lambda)\, d\lambda \tag{2.7}$$

$$\psi_1(\lambda) = [(1 + 2\lambda - \exp(-2\lambda)] / \Delta_+ \quad (2.8)$$

The conditions of boundedness of the operators L_1 and L_2 in R are

$$\| L_1 A \| < q \| A \|, \quad \| L_2 A \| < \varepsilon \| A \| \quad (2.9)$$

where

$$\varepsilon < 0.282\,(1-\mu)^2 / (3-4\mu) < 0.094 \quad (2.10)$$

and the inequality (2.4) holds for q. When the conditions (2.3), (2.4), (2.9) and (2.10) hold, an iteration method described in [5] can be applied to the operator equation (2.1), as follows:

$$A_{r+1} = L_1 A_{r+1} + L_2 A_r + d, \quad r \geqslant 0 \quad (2.11)$$

and we have

$$\| A - A_{r+1} \| < f \| A_r - A_{r+1} \|, \quad f = \frac{\varepsilon}{(1-q)}, \quad r \geqslant 0 \quad (2.12)$$

The following relation (in which A_0 is an arbitrary element) can be obtained

$$\| A - A_r \| < f^r \| A_1 - A_0 \|, \quad A_0 \subset R \quad (2.13)$$

From (2.4) and (2.10) we have

$$f < 0.263 \quad (2.14)$$

The estimates (2.13) and (2.14) indicate the rapid convergence of the method. (For certain values of μ the estimate (2.14) is too high. Thus for $\mu = 0.317$ we have $q < 0.39$, $\varepsilon < 0.076$ and $f < 0.125$.)

Let us write (2.11) in the form

$$\Phi_0 = L_1 \Phi_0 + d, \quad \Phi_r = A_{r+1} - A_r, \quad A_0 = (0) \quad (2.15)$$

$$\Phi_r = L_1 \Phi_r + L_2 \Phi_{r-1}, \quad r \geqslant 1 \quad (2.16)$$

The solution of (2.11) can now be written in the form

$$A_{r+1} = \sum_{n=0}^{r} \Phi_n$$

Thus we reduced problem of solving (2.1) to that of solving the systems of equations (2.15) and (2.16) which differ from each other only in their independent terms. For this reason the investigation of the system (2.15), (2.16) can be reduced to investigation of a single equation of the same form as (2.15), under the condition that the independent term is known.

Before investigating Eq. (2.15), we shall formulate some auxiliary lemmas.

Lemma 1. The solution of the system (1.8) can be written in the form

$$A_{(k)} = C_k / \omega_k^{\varepsilon_1}, \quad \omega_k = k\pi, \quad \varepsilon_1 = 1/2$$

where C_k is a bounded solution of the completely regular system:

$$C_k = \sum_{m=1}^{\infty} C_m M_{km} \omega_k^{\varepsilon_1} / \omega_m^{\varepsilon_1} + d_k \omega_k^{\varepsilon_1}, \quad k \geqslant 1$$

The following estimate also holds:

$$\sum_{m=1}^{\infty} |M_{km}| \omega_k^{\varepsilon_1}/\omega_m^{\varepsilon_1} < \eta < 0.77$$

$$\eta = S_2 S_3 \left[\pi\nu(\nu+2)\cos\frac{\pi\varepsilon_1}{2}\right]^{-1}, \qquad S_2 = (\nu-1)S_1 +$$
$$\pi(1+\nu\varepsilon_1)\left[2\cos\frac{\pi\varepsilon_1}{2}\right]^{-1}, \qquad S_3 = \nu - \varepsilon_1 +$$
$$2(\nu+1)^{\varepsilon_1}\exp(-\nu-1)\frac{1+2\varepsilon_1/(\nu+1)}{\Gamma(1+\varepsilon_1)},$$

$$S_1 = 4\sum_{n=0}^{\infty} \frac{(n+1)(-1)^n \alpha^{-2n-\varepsilon_1-1}}{(2n+1+\varepsilon_1)(2n+3+\varepsilon_1)}$$

$$\alpha^2 = (1-\mu)/\mu$$

Lemma 2. Let $\Phi(c_1+it)$ be an analytic function defined in the region $-1 < c_1 < 1/2$ and let $|\Phi(s)|$ satisfy, in this region, the following conditions:

$$|\Phi(c_1 \pm it)| < At^{-\alpha}, \quad \alpha > 0, \quad 1 < t < \infty$$
$$|\Phi(c_1 \pm it)| < B + Et^{\beta}, \quad \beta > 0, \quad 0 < t < 1$$

Then the relation

$$\sum_{m=1}^{\infty} \int_{c_1-i\infty}^{c_1+i\infty} \Phi(s)(\pi m)^{\gamma-s-1} ds = \int_{c_1-i\infty}^{c_1+i\infty} \Phi(s) \sum_{m=1}^{\infty} (\pi m)^{\gamma-1-s} ds$$

holds for all $\sigma < \operatorname{Re}\gamma < c_1$. The following estimates are used in proving the lemma:

$$|z_m| < (A+E)(\pi m)^{-c_1}(\pi\ln\pi)^{-1}$$
$$0 < S_N < (A+E)\pi^{-2-c_1+\sigma}(\ln\pi)^{-1}[(c_1-\sigma)^{-1} +$$
$$(N+1)^{-1}](N+1)^{-(c_1-\sigma)}$$

$$z_m = (2\pi i)^{-1}\int_{c_1-i\infty}^{c_1+i\infty} \Phi(s)(\pi m)^{-s} ds, \qquad S_N = \left|\sum_{m=N+1}^{\infty} (\pi m)^{\gamma-1} z_m\right|$$

Lemma 3. The equation

$$\Delta = 2\varkappa\cos\pi\gamma - 4\gamma^2 + 1 + \varkappa^2 = 0, \ \varkappa = 3 - 4\mu, \ \mu > 0.073 \qquad (2.17)$$

has in the region $\sigma > 0, \tau > 0$ ($\gamma = \sigma + i\tau$) a unique real root $1/2 < \sigma_0 < 1$, while the remaining roots are complex and form an enumerable set. In addition,

$$\operatorname{Re}\gamma_n = \sigma_n = 2n - \varepsilon_n, \ 0 < \varepsilon_n < 1/2, \quad n = 1, 2, 3, \ldots$$

3. Let us now investigate (2.15), rewriting it in the following form:

$$A_{0(k)} = \sum_{m=1}^{\infty} E_{km} A_{0(m)} + d_k, \qquad k \geqslant 1 \qquad (3.1)$$

The following theorem holds.

Theorem. At large values of k the solution of (3.1) can be written in the form of an asymptotic expansion

$$A_{0(k)} = \sum_{n=0}^{N} (a_n \omega_k^{-\gamma_n} + \ddot{a}_n \omega_k^{-\bar{\gamma}_n}) + R_N, \ \operatorname{Re}\gamma_n > 0 \qquad (3.2)$$

where γ_n are the roots of Eq. (2.17) and R_N is the remainder term for which the following estimates hold:

$$|R_N| < F_1\Gamma(2N+2)/(\omega_k c)^{2N+1}, \quad 0<c<1, \quad \omega_k c > 1 \tag{3.3}$$

$$|R_N| < F_2\Gamma(2N+2)/\omega_k^{2N+1}, \; c>1, \; \Gamma(N+1) = N\Gamma(N)$$

Here F_1 and F_2 are constants independent of k and N, and c is given by (1.2).

Let us assume that $A_{0(m)}$ in (3.1) are known. In this case, when k varies continuously over the range $0 < k < \infty$, the right-hand side of Eq. (3.1) is a known function continuous in $\omega_k = k\pi$. We denote this function by $\theta(\omega_k)$. By virtue of the fact that $A_{0(m)}$ is a solution of (3.1), the following relation holds for $k = 0, 1, 2, \ldots$:

$$\theta(\omega_k) = A_{0(k)} \tag{3.4}$$

and this implies that $\theta(\omega_k)$ continuously extends $A_{0(k)}$ into the region $0 < k < \infty$.

Let us set in (3.1) $\pi k = \omega$. Then we have

$$\theta(\omega) = \sum_{m=1}^{\infty} A_{0(m)} E_m(\omega) + d(\omega) \tag{3.5}$$

Let us apply the Mellin transform over the variable ω to both parts of Eq. (3.5), assuming that $\theta(\omega)$ refers to the class of functions for which this transformation exists. We denote

$$\Phi(\gamma) = \int_0^{\infty} \theta(\omega)\,\omega^{\gamma-1}\,d\omega, \qquad \gamma = \sigma + i\tau, \quad \sigma_1 < \sigma < \sigma_2 \tag{3.6}$$

$$K(\gamma) = \pi(\nu^2\gamma^2 - 1)/\nu(\nu+2)\cos^2(\pi\gamma/2), \qquad |\sigma| < 1 \tag{3.7}$$

$$D(\gamma) = \pi(1+\nu\gamma)\,[\Gamma(\gamma)\sin(\pi\gamma/2)\,c^{-\gamma} - I(\gamma)], \qquad |\sigma| < 1 \tag{3.8}$$

$$I(\gamma) = \int_0^{\infty} \lambda^{\gamma-1}\sin\lambda c\,\psi_1(\lambda)\,d\lambda \tag{3.9}$$

where $\psi_1(\lambda)$ is given by (2.8). Then (3.5) becomes

$$\Phi(\gamma) = K(\gamma)\sum_{m=1}^{\infty} A_{0(m)}(\pi m)^{\gamma-1} + D(\gamma) \tag{3.10}$$

It can be shown that Lemma 1 also holds for the system (3.1). Therefore the sum in (3.10) exists for all $\sigma < {}^1/_2$ and $\Phi(\gamma)$ is analytic in the region $-1 < \sigma < {}^1/_2$. Let us investigate the behavior of the function $\Phi(\gamma)$ in the region $\sigma > -1$. From the formulas (3.4), (3.6) and (3.10) we obtain, using the inverse Mellin transform,

$$A_{0(m)} = (2\pi i)^{-1}\int_{c_1-i\infty}^{c_1+i\infty} \Phi(s)(\pi m)^{-s}\,ds, \quad -1 < c_1 < {}^1/_2 \tag{3.11}$$

From (3.10) it follows that $\Phi(s)$ satisfies the conditions of Lemma 2. We therefore substitute (3.11) into (3.10) and apply Lemma 2 to find, that

$$\Phi(\gamma) = (2\pi i)^{-1}K(\gamma)\int_{c_1-i\infty}^{c_1+i\infty} \Phi(s)\,\pi^{\gamma-1-s}\,\zeta(1+s-\gamma)\,ds + D(\gamma) \tag{3.12}$$

$$-1 < \sigma < c_1, \qquad -1 < c_1 < {}^1/_2$$

where $\zeta(1+s-\gamma)$ is the Riemann zeta function [6].

Let us now investigate Eq. (3.12). We know that in the neighborhood of the point $s=\gamma$ the function $\zeta(1+s-\gamma)$ can be written as [6]

$$\zeta(1+s-\gamma)=(s-\gamma)^{-1}+F(s-\gamma) \tag{3.13}$$

where $F(s-\gamma)$ is a function analytic in the neighborhood of $s=\gamma$.

In order to study the behavior of (3.12) in the region $-1<\sigma<\infty$ we perform, in the integral appearing in the equation, a shift along s to the left of the line of integration $-1<c_1<{}^1/_2$. The resulting transformations yield

$$\Phi(\gamma)=4\pi\varkappa\frac{\nu^2\gamma^2-1}{\nu(\nu+2)\Delta}S(\gamma)+\frac{1}{\Delta}\varkappa D(\gamma)\cos\frac{\pi\gamma}{2} \tag{3.14}$$

$$S(\gamma)=(2\pi i)^{-1}\int\limits_{c_2-i\infty}^{c_2+i\infty}\Phi(s)\pi^{\gamma-1-s}\zeta(1+s-\gamma)ds \tag{3.15}$$

$$c_2<\sigma<{}^1/_2,\qquad -1<c_2<{}^1/_2$$

Here $D(\gamma)$ is obtained from (3.8) and Δ from (2.17). It is clear that (3.14) continues (3.12) analytically into the region $\sigma>c_2$. We can obtain the expression (3.2) from the formulas (3.11) and (3.14). Here $S(\gamma)$ is determined for all $\sigma>c_2$ by means of the relations (3.15) and (3.10), where $A_{0(m)}$ is the solution of (3.1). As the result the coefficients of the expansion (3.2) can be expressed, in particular, in the form of sums

$$\sum_{m=1}^{\infty}A_{0(m)}(\pi m)^{-2l-2}(\ln\pi m)^b,\qquad b=0,1,\quad l=0,1,\ldots,L \tag{3.16}$$

by displacing the line of integration in (3.15) to the left by $s=2L+2$.

When $l\geqslant 1$, only a few terms in (3.16) suffice. For $l=0$ the number of terms increases appreciably. At the same time the integral of the type (3.15) can be easily computed along the new line of integration. Using Lemma 3 we can express the remainder term as the following contour integral:

$$R_N=(2\pi i)^{-1}\int\limits_{c_N-i\infty}^{c_N+i\infty}\Phi(\gamma)\,\omega_m^{-\gamma}d\gamma,\qquad c_N=(2N+1) \tag{3.17}$$

Setting in (3.17) $\gamma=c_N+ip$ and using (3.14) and (3.10) we obtain, after lengthy manipulations, the estimate (3.3).

4. Numerical computations were performed for the case $\mu=0.31741$, $\sigma_0=0.70000$ and $c=0.5$ and 1. The systems (1.8) and (2.15) were solved by the reduction method. 20-th order solutions of (1.8) and (2.15) were found to differ from the corresponding 30-th order solutions by not more than 3.5%. The estimates (2.12), (2.10) and (2.4) indicate that for all $\mu<0.31741$ the quantity in question varies insignificantly. Thus in the engineering practice the solution of the less cumbersome system (2.15) can be used whenever $\mu\leqslant 0.31741$.

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ON THE STATE OF STRESS AND STRAIN IN A FINITE CYLINDER SUBJECTED TO DYNAMIC LOADS

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A solution is presented of the dynamical axisymmetric problem of elasticity theory for a cylinder of arbitrary length with given displacements on its curved and planar surfaces. The initial non-self-adjoint equations are converted into equivalent first order equations for an extended eigenvector by introducing certain auxiliary functions. Arbitrary displacements given on the flat endface of the cylinder are expanded in series of eigensolutions of the problem by using these eigenvectors. Final formulas are obtained for the expansion coefficients. As a particular case, the solution of the statics problem of a cylinder [1] follows for $\omega \to 0$. An analogous problem has been examined in [2] where it was reduced to solving an infinite system of equations. The numerical method for solving problems of such a class has been elucidated in [3].

1. Let us proceed from the differential equations in displacements

$$v_1^2\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+v_2^2\frac{\partial^2 u}{\partial z^2}+ (v_2^2-v_1^2)\left(\frac{\partial^2 w}{\partial z\partial r}+\frac{1}{r}\frac{\partial w}{\partial z}\right)-\frac{\partial^2 u}{\partial t^2}=0 \tag{1.1}$$

$$v_2^2\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}\right)+v_1^2\frac{\partial^2 w}{\partial z^2}+(v_2^2-v_1^2)\frac{\partial^2 u}{\partial z\,\partial r}-\frac{\partial^2 w}{\partial t^2}=0$$

$$v_1^2=\frac{\mu}{\rho},\qquad v_2^2=\frac{\lambda+2\mu}{\rho}$$